

DETERMINING THE IMPACT OF OVERLOAD ON MARKOV PROCESSES IN THE QUEUEING SYSTEM BASED ON AN ANALYSIS OF CUSTOMERS PATIENCE IN QUEUE

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Abstract

The impact of overloading the queueing system on Markov processes was based on an analysis of customers patience in queue. In this paper the regimes of the unloaded and overloaded operation of the queueing system are observed. In the case of overloading, there is a general change in Markov characteristics. Implications for patient and impatient customers are in queue with the significance. Customer patience was degraded quantitatively and qualitatively in overload mode of operation. These characteristics are confirmed on the concrete example and measurements of parameters of the queueing system - parking garage.

Keywords - *queueing theory; parking garage; customers patience; overload*

INTRODUCTION

Customers patience is a significant parameter of the queueing theory. The structure of customers in the system is heterogeneous, i.e. there are *patient customers*, which were waiting in the queue for the service, regardless of the length of time they had been waiting for, and *impatient customers*, who after spending time in the queue, left the queue and thereby cancel services.

The emergence of a large process of impatient customers gives the perception of a low quality of service, causes loss of customers [1] and negative economic results. The researches of impatience of customers dominate in telecommunication systems [2] and in the research of the parameters of patience [3, 4, 5].

Researching customers patience in stationary traffic - parking is modest literary processed. The first research has established a significant participation of the flow of impatient customers and established non-homogeneous causes of impatience.

Based on the second analysis of the customers patience, it was established that in case of overloading the system, the Markov concept of work is not sustainable, and therefore the optimization of the system can not be carried out using the existing analytical methods. In case of overloading the system, it is also found to be significant participation of the flow of impatient customers in the total flow, as well as the causes of impatience and their homogeneity.

METHODOLOGY OF RESEARCH

In this paper we examined the impact of overloading on Markov's processes in the queueing system based on the analysis the time that customers (patient and impatient) spend in queue waiting for the service.

These analyzes were carried out on the real system - parking garage in case of unloaded and overloaded work of queueing system. The first analysis of customers patience was carried out on 28.06.2017. in case of unloaded work of queueing system [7]. The second research of customers patience in case of overloaded work was carried out on 05.06.2018. in the period from 7:00 to 21:00 at the parking garage "Serbian National Theater" (SNT) in the city of Novi Sad.

Methodology of customers patience research was included recording the following times: the time of arrival of the customer in queue, the time of entering the customer into the parking garage and the time of the customer cancellation. These times are important for getting the parameters of customers patience.

Also, the paper presents analysis of the incoming flow of customers and analysis of the retention time of customers in the system.

QUEUEING SYSTEM OF PARKING GARAGE

The observed parking garage SNT is located in the central zone of Uspenska Street in Novi Sad. The construction of the parking garage is carried out in the system "fast park" above the already existing parking lot.

The static capacity of this parking garage is 104 parking spaces. The vehicles are parked at an angle $\alpha=90^\circ$ and at an angle $\alpha=0^\circ$. Other characteristics of the observed parking garage are shown in the paper [7].

The parking garage SNT is a classic Erlang queueing loss system, FIFO discipline services and Kendall's notation $M(\lambda_{pq})/M(\mu)/104/10+M(\lambda_{iq})$. It

has been accepted that the parking service has “104” service channels and “3+7” spaces in a queue (respectively regular + irregular).

The initial operating time of the system is characterized by a negligible number of vehicles, that do not affect the systems ergodicity.

GPS Coordinates: N 45° 15' 16.2" E 19° 50' 31.92"

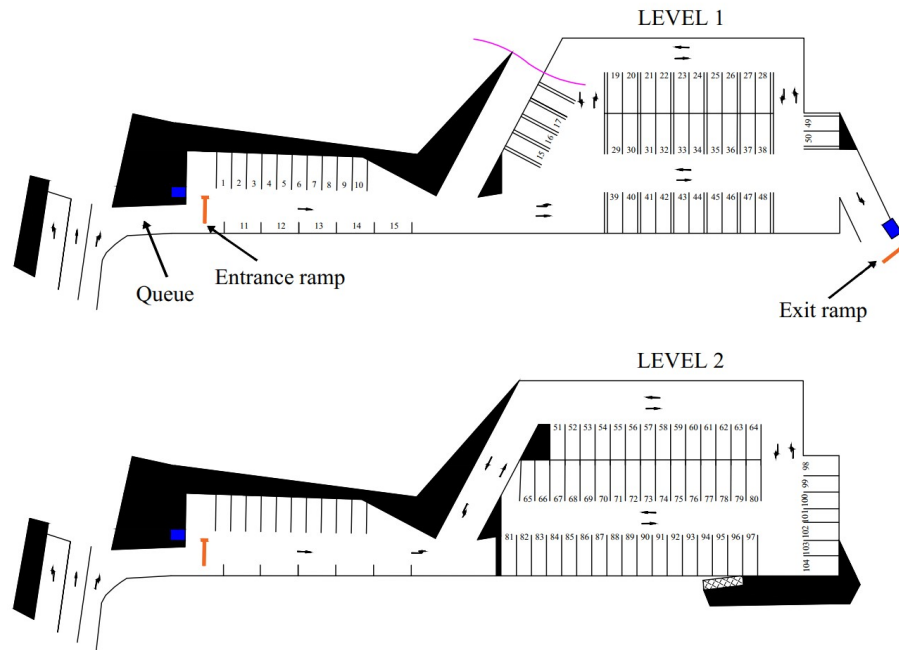


Fig. 1. Queuing system - parking garage SNT

Thereby, the parking garage has the capacity that absorbs the first hours of peak loads (104 service channels) and average service time from $\mu=0.3765$, i.e. an average service time from about 2.65 h/vehicle. The customer service time on the ramp is an average of 30 seconds [7].

INPUT FLOW ANALYSIS

The analysis of the incoming flow of customers implies determining the legality of the distribution of the probability of the interval between the moment of arrival of customers (passengers, vehicles, etc.) or the legality of distribution of the probability of arrival of a certain number of customers (passengers, vehicles, etc.) at a given interval.

Table 1 shows the basic parameters of the input flow in certain years of observation, on the basis of which it is possible to assume the law of the distribution of the input flow.

Table 1. Basic parameters of the input flow in certain years of observation

Years of observation	Mean value $M(X)$	Dispersion $D(X)$	Ratio $M(X)/D(X)$	Chi-Square test	Threshold of significance p	Poisson input
2010	11,89	23,28	0,51	11,45	0,02193	No
2013	10,73	21,29	0,50	10,44	0,03353	No
2017	9,64	17,76	0,54	18,61	0,00033	No
2018	11,49	14,49	0,81	4,073	0,25360	Yes

Since the parameters refer to the number of customer arrivals for the selected interval, it is necessary to determine the quotient of mean value and dispersion. In the period from 2010 to 2017, this ratio is less than 1, which means that customer arrivals do not behave according to Poisson's distribution. In order to confirm this, it is necessary to test the hypothesis of the distribution law.

Verification of the mean number of vehicles that joined the service was performed χ^2 test, for the adopted significance threshold $p=0,05$. On the basis of the performed testing, the software failed to verify the Poisson distribution for the adopted significance threshold (0.05), but for $p < 0.05$, which means that we have a basis to reject the hypothesis, which leads us to conclude that the mean number of vehicles joined the service in the period from 2010-2017 does not behave according to Poisson's distribution. In the year 2018, the Poisson distribution for the significance threshold $p=0,25360 > 0,05$ was verified, which means that we have no reason to reject the verified hypothesis, which leads us to conclude that the average number of vehicles that joined the service is behaving Poisson's distribution.

ANALYSIS THE RETENTION TIME OF CUSTOMERS IN THE SYSTEM

In applying the queueing theory, one can not always accept the assumption that the service time has an exponential distribution [8]. With this in mind, it is necessary to perform testing of customer retention time in the system.

Basic parameters of customer retention time in the system was given in Table 2.

Table 2. Basic parameters of customer retention time in the system

Parameters	Mean value $M(T)$	Dispersion $D(T)$	Standard deviation $\sigma(T)$	Coefficient of variation ν	Erlang parameter k
Value	95,2	4773,2	69	0,72	1,9

Since the Erlang parameter is $k=1,900 \approx 2$, it is necessary to check the agreement of the empirical distribution with Erlang's second order distribution ($k=2$).

Vehicle retention time on the parking garage is classified into 24 classes, a 15 minute interval, which is important to prove that the vehicle's retention time has Erlang's distribution.

Since the number of classes is 24, the degree of freedom is $n = r - 2 - 1 = 24 - 2 - 1 = 21$. It was found that the retention time had Erlang's distribution, and to prove this χ^2 test was made. Erlang's distribution with a degree of freedom 21 and a 5% risk has $\chi^2 = 32.67$. This value is greater than the calculated value $\chi^2 = 26.98$, so there is no reason to reject the verified hypothesis (shown at Figure 2).

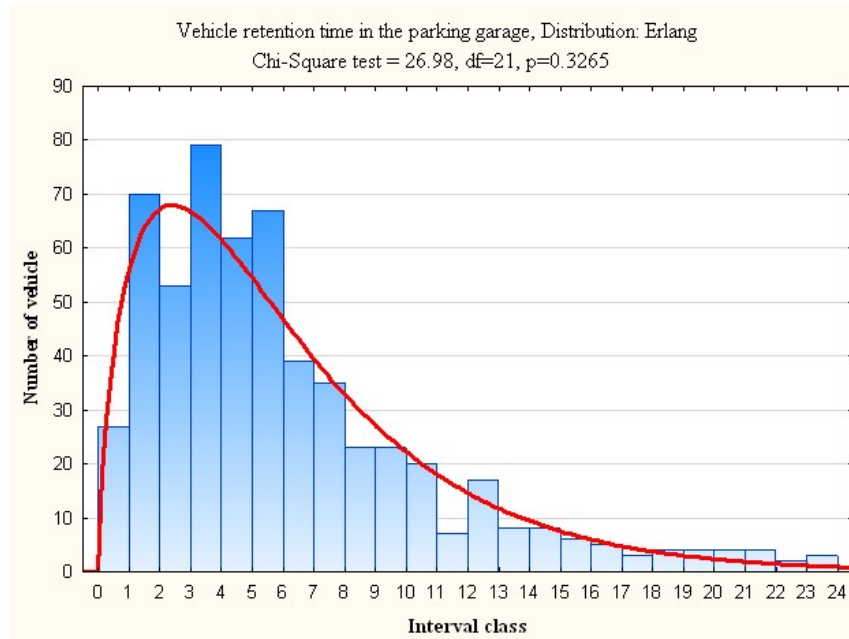


Fig. 2. Verification of the Erlang distribution of vehicle retention time in the parking garage

THE CASE OF UNLOADED SYSTEM OPERATION

Analysis of customers patience was carried out in paper [7]. Based on this research, we can conclude that the parking garage was unloaded in the observed period. It was noted that there are four peak periods in which the customers spend most of their time waiting for the service (shown at Figure 3).

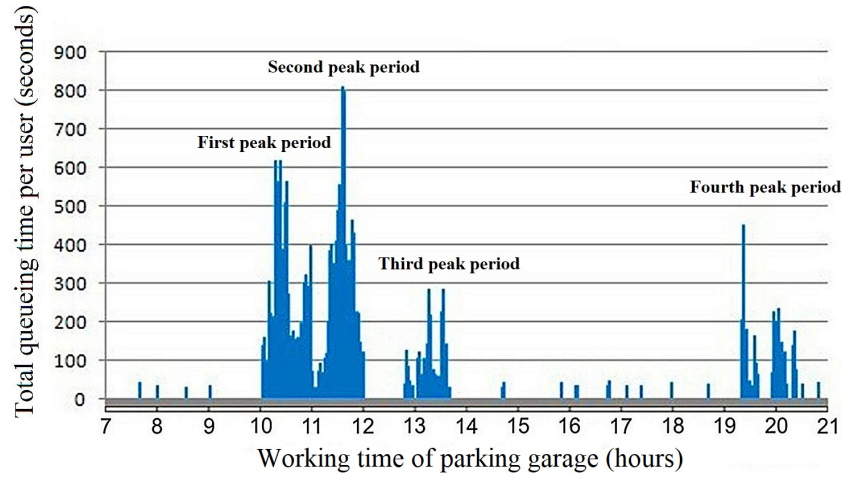


Fig. 3. Waiting time of customers in queue in certain hours of parking operation, [7]

Figure 3 shows the maximum and minimum waiting time of the customers in a queue. Maximum waiting time of the customers in a queue occurs between 11:00-12:00 (808 seconds), and at least between 12:00 and 13:30 (274 seconds).

Verification of waiting time of customers in queue in case unloaded system

The total daily arrival rate of the parking garage was 568 vehicles. Average 207 vehicles came into from the queue. This process is differentiated into the: *patient* (143 customers) and *impatient* (64 customers).

We can conclude that 30.92% of customers who have joined the queue are distributed to the impatient customers.

The times that customers spend in queue are exponentially distributed for both customer processes. Verification of the average time spent by customers in a queue is done by χ^2 test for the adopted threshold of significance $p=0.05$. For the patient customers, the exponential distribution

is verified with $p=0.71547$, and for the impatient customers with $p=0.07624$ [7].

For the patient customers who waited for the service, the average time spent on the queue was up $t_{qp}=188.11$ sec ($\lambda_{qp}=0.005316$) and for impatient customers $t_{qi}=87.39$ sec ($\lambda_{qi}=0.01144$) [7].

THE CASE OF OVERLOADED SYSTEM OPERATION

On the basis of the second research, it can be concluded that the parking garage was overloaded in the observed period and there are four peak periods in which the customers spend most of their time waiting for the service (shown at Figure 4).

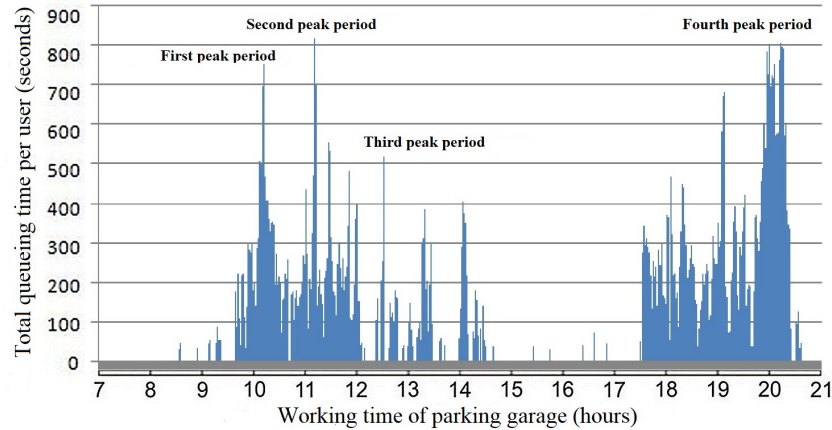


Fig. 4. Waiting time of customers in queue in certain hours of parking operation in case of overloaded operation of the system

Figure 4 shows that during the system overload, the queues do not appear in the period from 7:00 to 9:40, due to filling the parking garage. In the first peak period (9:40-10:30) it can be noticed that the time that the customer spends in queue it starts to grow until the maximum is 750 seconds. The second peak period begins immediately after the first (10:30-12:00) in which the time the customers spends in queue after the reduction is again increased to maximum value of 803 seconds. The third peak period occurs in the period from 12:35 to 14:30, where this time begins to drastically decrease. The maximum value of this time in the third peak period is 517 seconds. The fourth peak period occurs in the period from 17:30 to 20:20, where the maximum waiting time of the customers in a queue is 804 seconds.

Verification of waiting time of customers in queue in case overloaded system

The total daily arrival rate of the parking garage was 644 vehicles. Average 453 vehicles came into from the queue. This process is differentiated into the: *patient* (324 customers) and *impatient* (129 customers). We can conclude that 28.47% of customers who have joined the queue are distributed to the impatient customers.

Verification of the average time spent by customers in a queue is done by χ^2 test for the adopted threshold of significance $p=0.05$. For the patient customers, the Gama distribution is verified with $p=0.09407$ (shown at Figure 5).

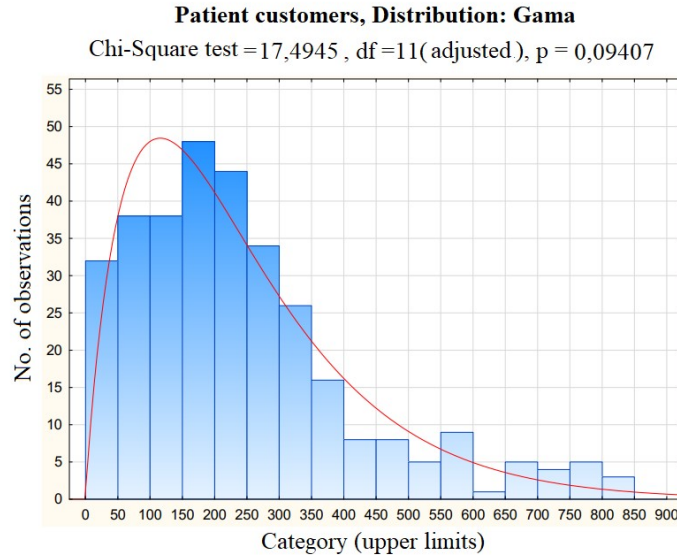


Fig. 5. Verification of the Gama distribution of patient customers in queue ahead of parking garage in case of overloaded operation of the system

For the impatient customers, the exponential distribution is verified with $p=0.05354$ (shown at Figure 6).

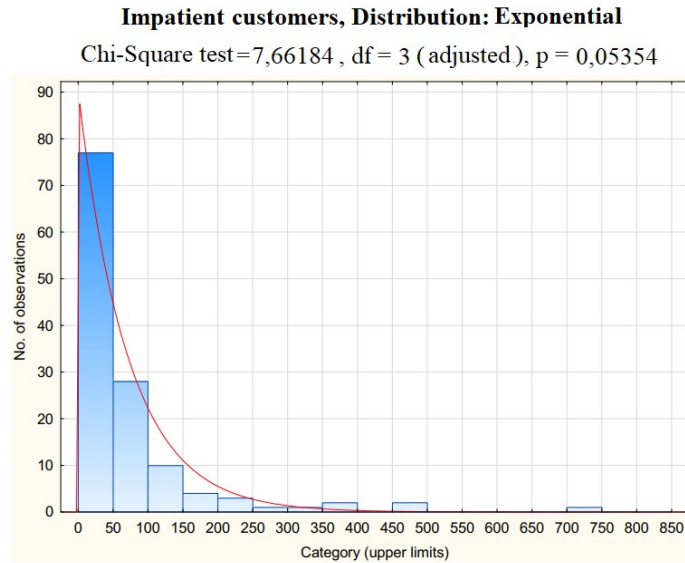


Fig. 6. Verification of the exponential distribution of impatient customers in queue ahead of parking garage in case of overloaded operation of the system

For the patient customers who waited for the service, the average time spent on the queue was up $t_{qp}=244.30$ sec ($\lambda_{qp}=0,00409$) and for impatient customers $t_{qi}=71,60$ sec ($\lambda_{qi}=0,01396$).

QUANTITY AND QUALITY OF IMPATIENCE OF CUSTOMERS

Quantity was related to participate in the flow of impatient customers in the total flow, and the quality of impatience was related to the causes and their homogeneity. The research has established a significant participation of the flow of impatient customers and established non-homogeneous causes of impatience in both cases of work (unloaded and overloaded system operation).

Table 3 shows distribution of impatient customers in case of unloaded system work [7].

Table 3. Distribution of impatient customers in case of unloaded system at hourly intervals, [7]

	Morning peak periods			Afternoon peak periods	
Intervals	10-11	11-12	12-13	18-19	19-20
Number of vehicles	2	9	14	23	16
ΣP (100%)	3,13	14,06	21,88	35,94	25
Input process	28	40	56	44	44

It is interesting to notice that in the morning peak periods it occurs a smaller process of impatient customers 39.06% and in the afternoon complementary 60.94% of the impatient customers.

All impatient customers are recorded in two intervals (from 10:00 to 13:00; from 17:00 to 20:00). The structure of the phenomenon of patience is obviously greater with the condition that the parking requirement is due to work obligations (the morning requests). In the afternoon peak periods, these requests have been changed to the other contents: an access to the theater or cinemas, or intent of consuming the other contents of the city center.

It's interesting to note (shown at Figure 3) that in a afternoon peak period a large number of the impatient customers appeared. Such a distribution probably follow from the subjective attitude of time in different circumstances. Established process of impatient customers is contained average $64/568=0.1126$, or 11.26% customers canceled the service from queue [7].

Table 4 shows distribution of impatient customers in case of overloaded system.

Table 4. Distribution of impatient customers in case of overloaded system at hourly intervals

	Morning peak periods			Afternoon peak periods		
Intervals	10-11	11-12	12-13	17-18	18-19	19-20
Number of vehicles	29	17	12	8	33	30
ΣP (100%)	22,48	13,18	9,30	6,20	25,58	23,26
Input process	48	38	44	66	49	52

From Table 4 it can be noticed that in the case of overloaded system operation in the morning peak periods, the flow of impatient customers is 44.96%, and in the afternoon complementary 55.04% of impatient customers.

In case of system overloading, the flow of impatient customers doubled (129 customers) compared to the case when the system was unloaded (64 customers). Established process of impatient customers is contained average $129/644 = 0.2006$, or 20.03% customers canceled the service from queue.

Also, we do not know the structure of another process - discouraged customers. These are potential customers who left the intention to park when they spotted the length of the queue at the entrance ramp. They can be absorbed in the event of an increase in the quality of the service.

CONCLUSION

Based on the research carried out, it can be concluded that in both cases of system operation there are four peak periods in which customers spend most of their time waiting in queue for a service. It is interesting to note that in the case of a overloading operation of the system, the fourth peak period begins much earlier and there is a large number of patient customers in it, while in the case of unloaded work it was not the case. It is also important to note that the maximum waiting time for customers in the queue at peak periods has increased in the event of an overloaded system operation.

In Markov's queueing systems, the average time that customers spend in queue is exponentially distributed. The analysis found that the time that patient customers spend in queue, in the case of unloaded system, are distributed according to the exponential distribution, while in the case of overloaded system distributed by the Gama distribution. We can conclude that the analyzed parking garage can be viewed as the Markov system only in the case of an unloaded system, when the average time that customers spend in the queue, exponentially distributed. In case of overloading the system, the average time spent by customers in a queue is distributed according to the Gama distribution, which leads to the conclusion that the observed parking garage can not be regarded as the Markov system.

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